REDISTRICTING WITH OPTIMIZATION

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Redistricting..

US Census Bureau does a full census every ten years

Must use to redraw district boundaries *i.e. redistrict*, each state and city for the election of representatives to

- US Congress
- State Senate
- State House
- City Wards, etc.

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..So that

The districts are balanced and fair

Make geographical sense

Supported by consensus

Maybe don't want…

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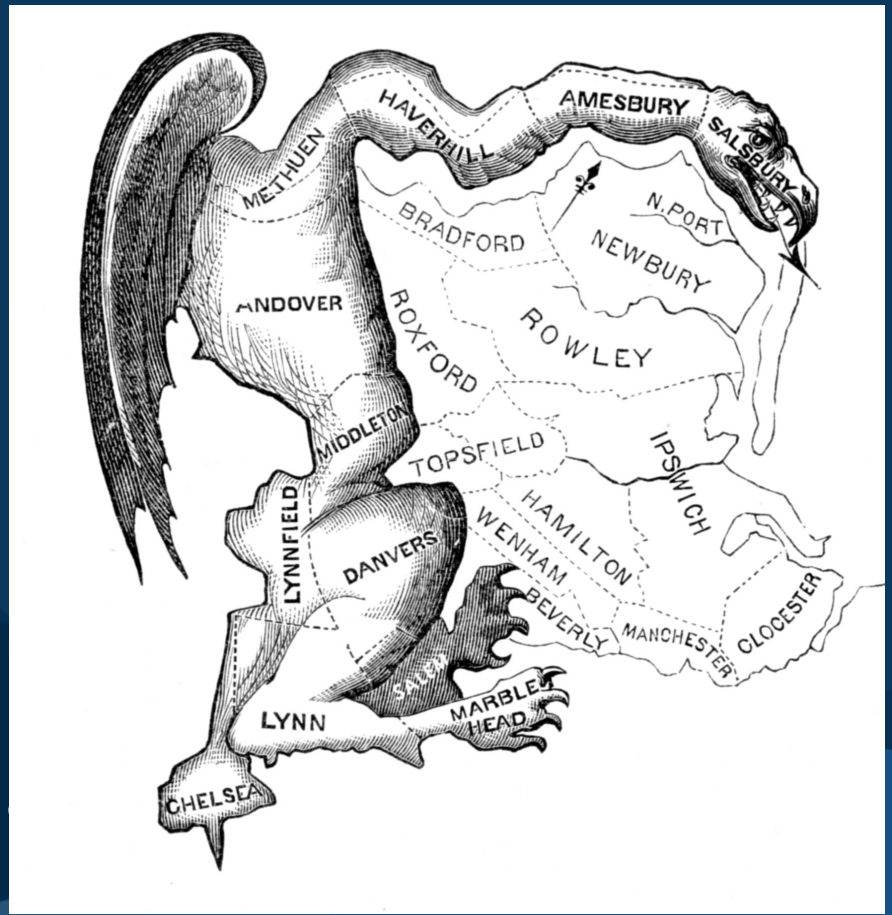
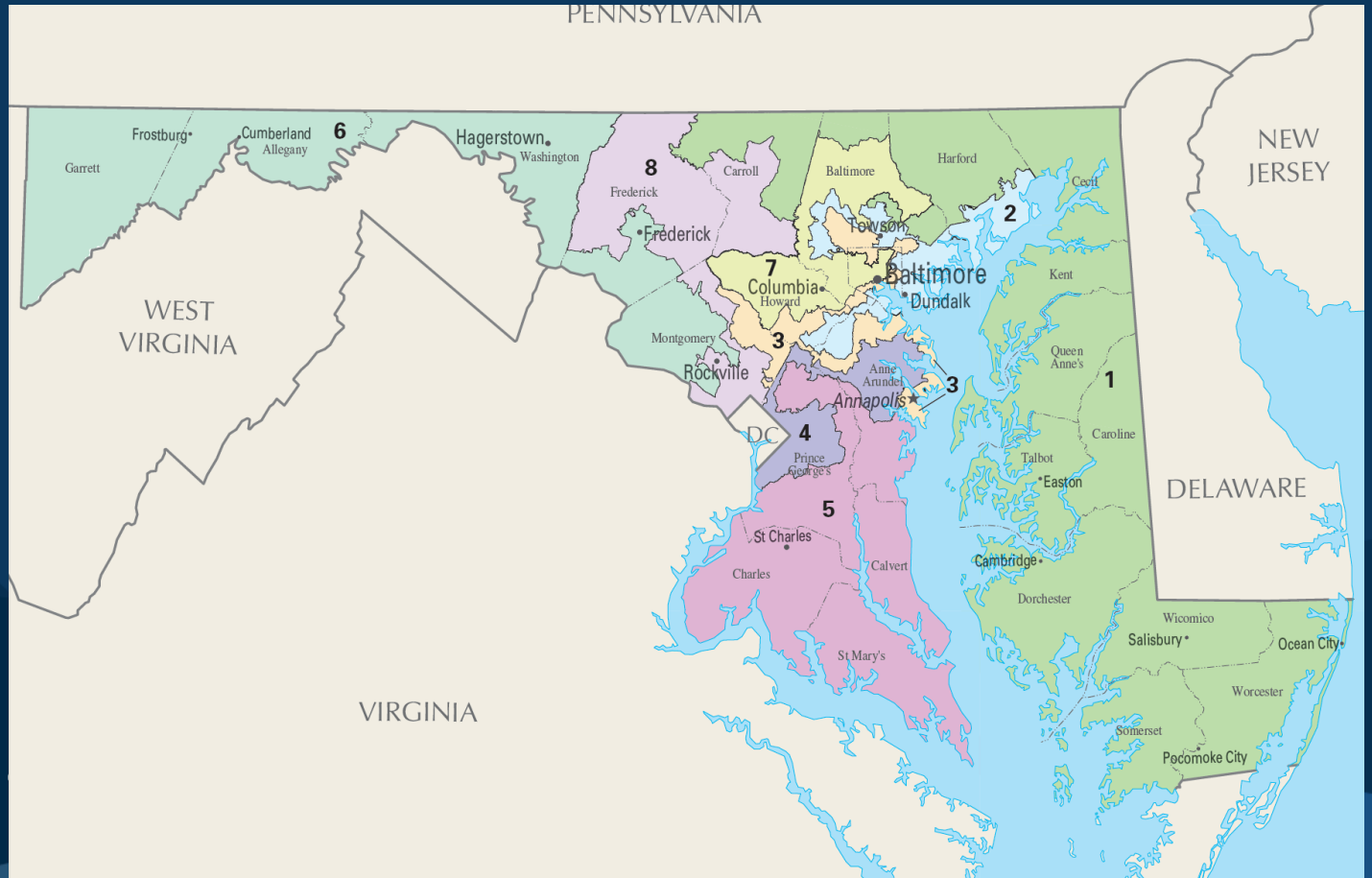
Boston in 1812



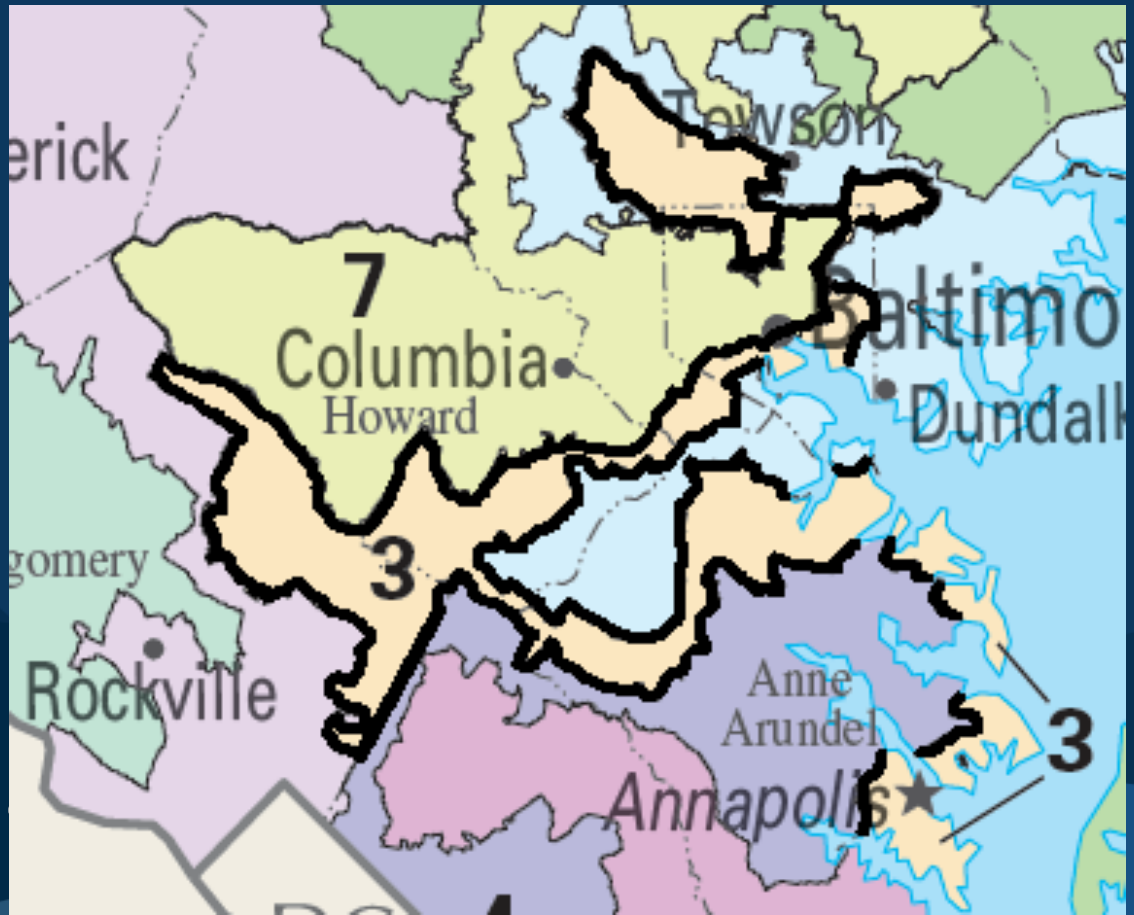
South Essex district

Created by Massachusetts governor Elbridge Gerry

Maryland Congressional Districts (2010 Census)



Maryland Congressional District 3



..So that

The districts are balanced and fair

Make geographical sense

Supported by consensus

Maybe don't want…

- Argument / Dispute
- Litigation
- Delay

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Need

Procedure for drawing district boundaries that is:

- Flexible
- Transparent
- Auditable
- Beyond dispute

What about optimization?

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Optimization

Mixed Integer Programming (MIP)

"optimization .. for full-scale districting plans are likely computationally intractable .." DeFord et al. (2021)

".. literal global optimization is completely intractable for problems of this size and complexity.." Duchin (2021)

I would agree… until now

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Redistricting: The Task

US Census Bureau (Public Law 94-171) divides each State into divisions such as:

	Average Pop	No. in Virginia
Tract	4000	2198
Block Group	1500	5963
Voter District	2500	3531
Block	50	163491

Task is to assign each *division* to a *district*

Have districts such as

	range	No. in Virginia
Congressional	1 - 52	11
State Senate	20 - 67	40
State House	40 - 400	100

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Redistricting: The Constraints

Balance: the population of each district must be the same (+/- 2%) Objective then Hard

Contiguity: the districts cannot be split into separate geographical areas Hard

Compactness: the districts should be compact and not elongated or splattered and should not have holes Soft

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Redistricting: The Constraints 2

Minority-Majority Voting Rights Act (1965 and renewed): if a district with a majority of a minority (racial or community interest) can reasonably be created, it should be **Soft**

No unnecessary splits: counties e.g. should not be needlessly split across districts **Soft**

Proximity to previous districting: if it was a good one **Soft**

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Redistricting: The Math based on Hess et al. (1965)

Tracts $t \in \mathcal{T}$

Choose $t_1, t_2, \ldots, t_D \in \mathcal{T}$ to be tracts serving as centers of districts 1, 2, …, D

Decision variables:

$x_{td} \in \{0,1\}$ where $t, d \in \mathcal{T}$

1 iff tract t assigned to district centered on tract d

STO $x_{dd} = 1$ iff d is a district center

Assignment

Each tract must belong to a district

$$\sum_{d \in \mathcal{T}} x_{td} = 1 \quad \forall\, t \in \mathcal{T}$$

Assignment of tracts to districts

$$x_{td} \leq x_{dd} \quad \forall\, t, d \in \mathcal{T}$$

Need exactly N districts

$$\sum_{d \in \mathcal{T}} x_{dd} = N$$

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Have a MIP – what could go wrong?

Model Size

- Congressional districting:
  - 677K binaries 18M matrix elts for Arkansas
  - 5M binaries 129M matrix elts for Virginia

Poor results

- Shape of districts not good
- Could have holes i.e. a district being a ring around one or more others

Making MIP Work: Size

Remove unnecessary edges from assignment graph i.e. potential allocations

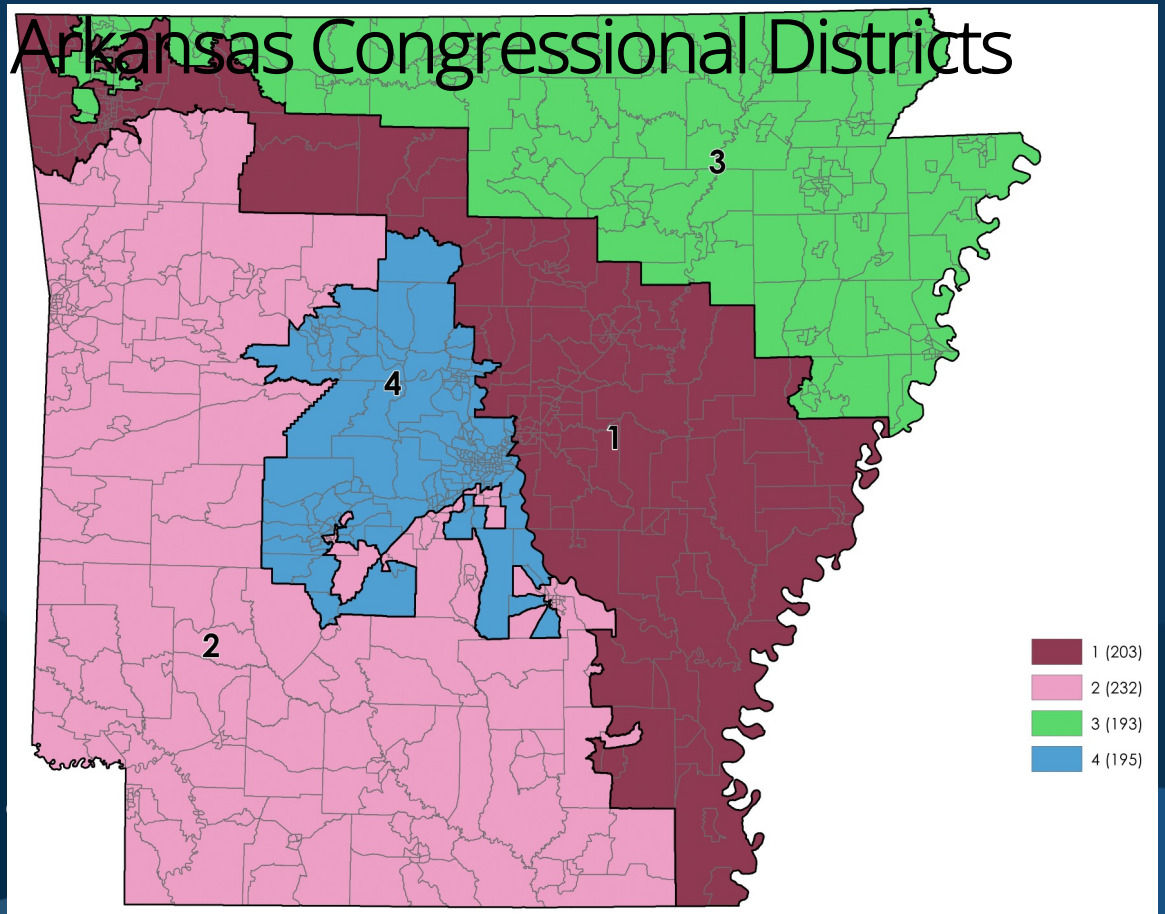
Can reduce the n(n-1) edges by 1% - 99%

Depends on number of districts as well as tracts

Only use subset of $\mathcal{T}$ as candidates for district centers

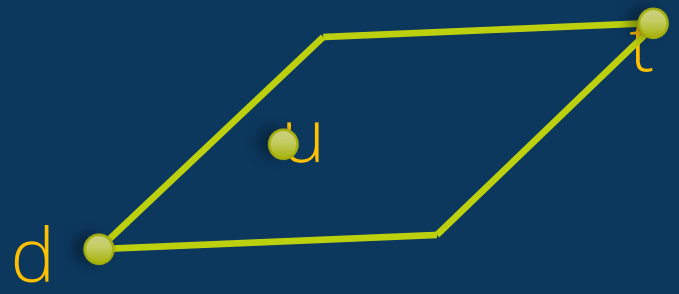
Fatuous to use all $\mathcal{T}$ as such candidates

Making MIP Work: Poor District Shapes



Making MIP Work: Poor District Shapes

Introduce diamond constraints



Bans holes

If t is assigned to district centered on d ensure all tracts with centers in the diamond e.g. u are also so assigned

Increases model size

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Arkansas Again

Diamond constraints

Penalize splitting capital city

Penalize splitting counties

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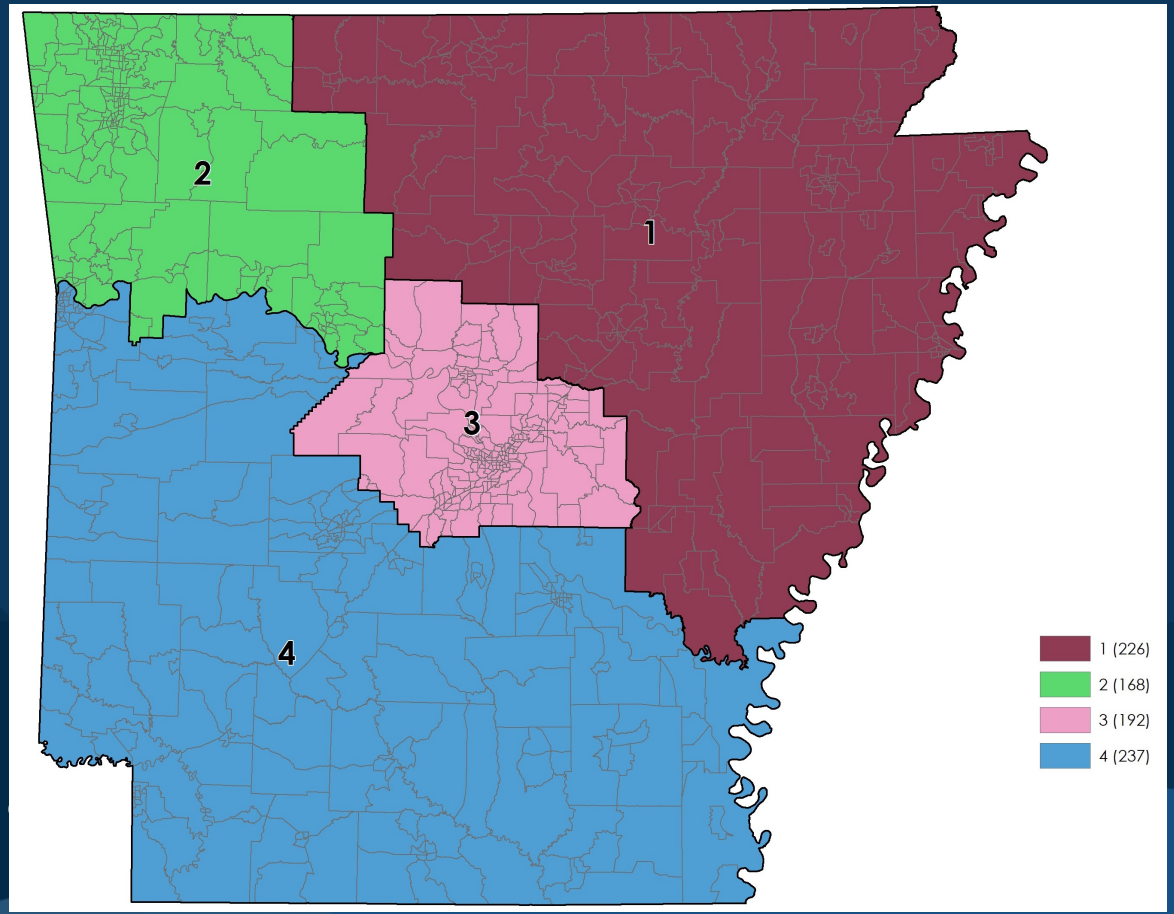
Arkansas Again



Keep Little Rock together in same district

No split counties

Maximum pop. dev. < 1.5%

Making MIP Work: Performance

Tighten contiguity constraints

Run a sequence of models using the solution from the last as a start for the next

Start with as few hard constraints as possible and minimize district population deviation
Leave out min-maj, splits etc. to begin with

Harden population balance constraints

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Making MIP Work: Performance 2

Add soft constraints for: Diamonds; Min-maj; Split counties; etc. in order of priority

Use increasing numbers of candidate district centers

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Making MIP Work: Performance 2

Use the most powerful large-scale optimizer:
ODH|CPLEX or ODH|Gurobi

Standard MIP optimizers will likely still fail

Get a sequence of improving solutions

Stop when relevant KPI achieved or time limit hit

Usually aim for ~ 5% optimality gap

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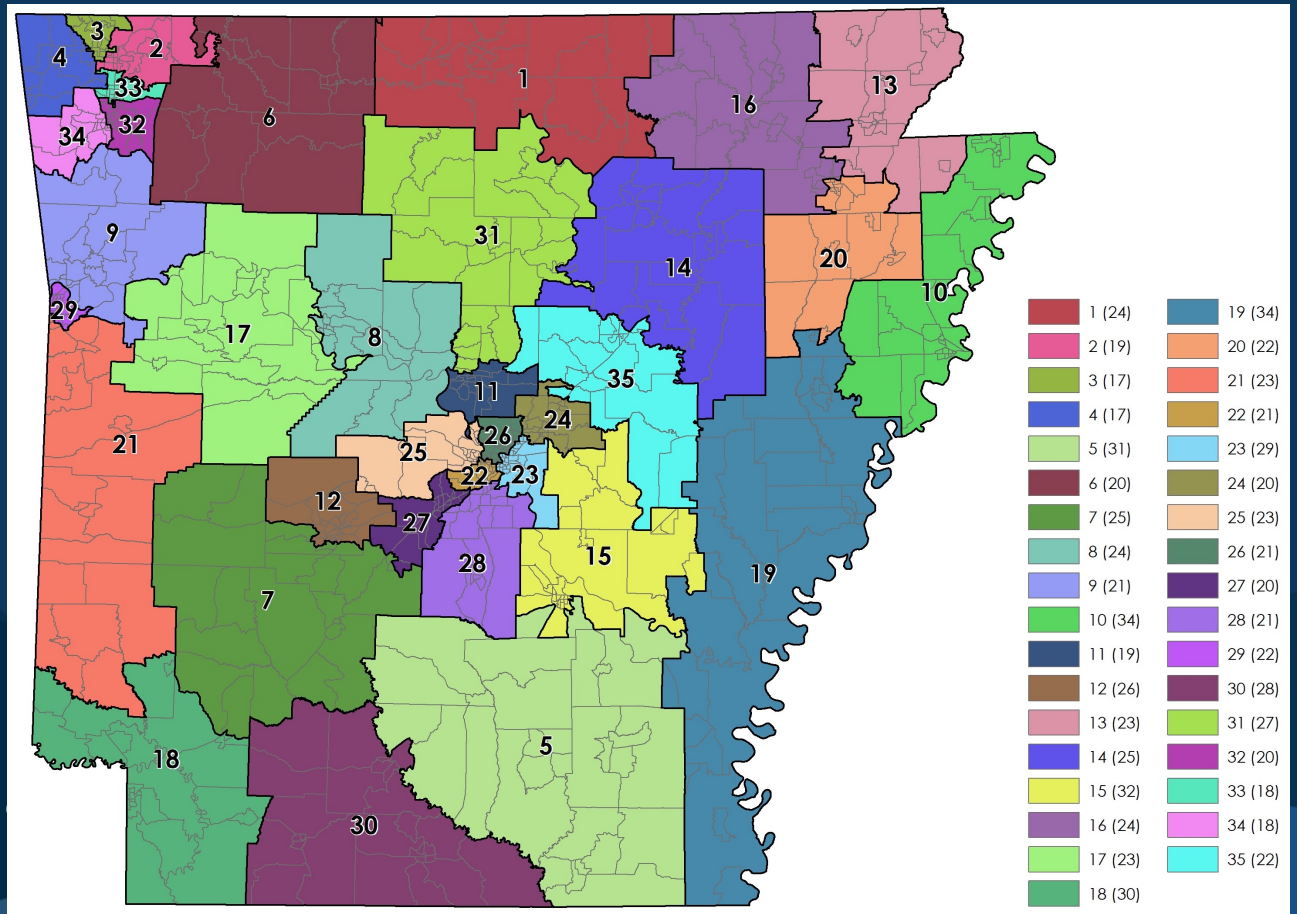
Arkansas State Senate



Max dev
2.67%

5 min-maj
districts

26 split
counties

Using MIP in Practise: The City of Pine Bluff

Pine Bluff is a city in Arkansas

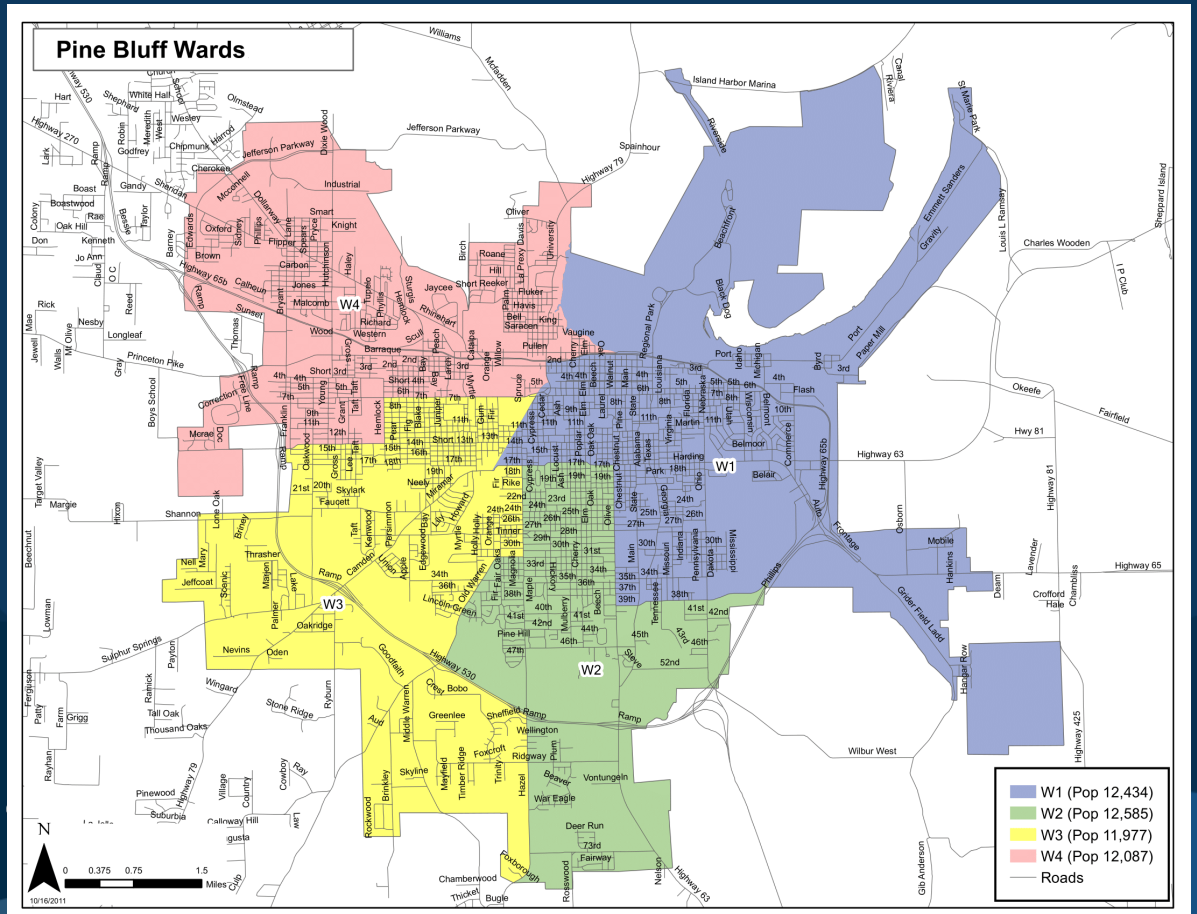
Divided into 4 wards

Population declined by 13% since 2010

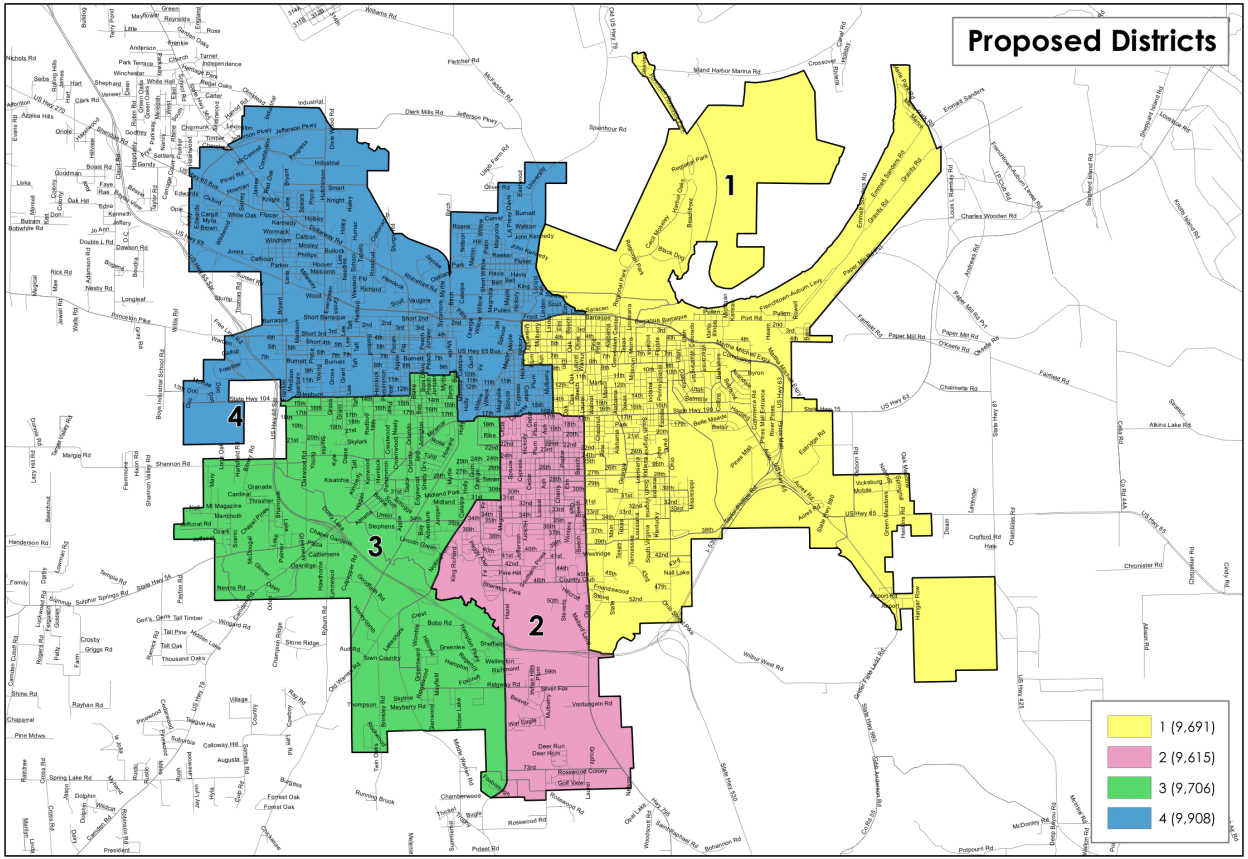
Need to redraw the ward boundaries

Divide the city into 109 voting districts

Pine Bluff in 2010



Final Redistricting



Conclusions

MIP is a useful tool for redistricting

All constraints except contiguity are soft

Flexibility offered by MIP is essential in practise

Established methodology, simple python model and commercial software $\Rightarrow$ **Auditable**

Must take care with modelling

Must use powerful large-scale optimizer like ODH

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References

DeFord D., Duchin M., and Solomon J. (2021), "Recombination: A Family of Markov Chains for Redistricting", *Harvard Data Science Review*, Issue 3.1

Duchin M. (2021), *by email* 7/9/21

Hess S.W., JB Weaver J.B., Siegfeldt H.J., Whelan J.N., and Zitlau P.A. (1965) "Nonpartisan political redistricting by computer", *Operations Research*, 13(6):998-1006.

Oehrlein J. and Haunert J-H. (2017), "A cutting-plane method for contiguity-constrained spatial aggregation", *Journal of Spatial Information Science*, (15):89-120.

ODH|CPLEX (2022), see www.optimizationdirect.com/ODheuristics.php

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Thanks for listening

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Making MIP Work: Additional Constraints

Use soft constraints to handle

- Minority-Majority, Splits and Proximity

Not complicated to do

- Had enough math already!
- But need to be careful in choice of penalties